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TECHNICAL NOTE 3719

APPLICATION OF SCATTERING THEORY TO THE MEASUREMENT OF
TURBULENT DENSITY FLUCTUATIONS BY AN OPTICAL METHOD

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SUMMARY

An analysis shows that the scattering of unpolarized plane light waves which penetrate turbulent, transparent gases provides a measure of the average integral scale and intensity of the turbulent density fluctuations. The analysis is based upon the scattering coefficient deduced by Booker and Gordon and, thus, assumes isotropic fluctuations having an exponentially decaying (Markoffian) correlation function. Photometric data taken through turbulent boundary layers in air are found to conform functionally with the analytical prediction.

INTRODUCTION

The effect of random variations in index of refraction of the atmosphere on the propagation of electromagnetic waves has recently received widespread theoretical consideration (refs. 1 to 3). Attention, in the main, has been concentrated on the problem of the "forward scattering" of vhf and uhf radio waves with the intent to explain unexpectedly high signal strengths which exist beyond the line of sight. In general, it has been demonstrated that this scattering of beamed radiation out of the line of sight can be explained by postulating reasonable values for the intensity and scale of turbulent density fluctuations in the atmosphere.

An analogous scattering occurs when a wave of visible light traverses a turbulent boundary layer, wake, or jet. The significant interest in the scatter phenomenon for light transmission is not reception beyond the line of sight, but the examination of the scattered light field with the intent of deducing the turbulent parameters that give rise to scattering.

Liepmann (ref. 4) studied the fluctuations in the deflection angle of a light ray passing through a turbulent boundary layer and obtained the mean square deflection in terms of the correlation function and the mean square fluctuations in refractive index. The results at radio wave lengths and Liepmann's analysis suggest that measurements in the scattered field can be applied to determine the turbulent properties of the medium.

The purpose of this report is to describe an optical method for measuring the average intensity and integral scale of turbulent density

fluctuations in transparent gases. The method relies upon the hypothesis that the turbulent field can be uniquely defined in terms of the scattered electromagnetic field which arises when a plane wave passes through the turbulence. Photometric exploration of the scattered field can then be interpreted with the aid of existing theory to yield the turbulence structure as characterized by the integral scale and average intensity of the density fluctuations. The application of forward scattering theory to such measurements was discussed in a paper by Stine and Winovich given before the joint meeting of the American and Mexican Physical Societies in Mexico City, August 31, 1955. This report is an expanded version of the paper.

The significance of the theory developed here is that it provides a means for measuring turbulent density fields by relatively simple optical methods. Although the technique has limitations, it has the singular advantage of responding only to density fluctuations as opposed to the mixed-response signal of the hot-wire anemometer. Thus, it promises to be especially applicable to turbulence measurements in compressible flows.

NOTATION

c	velocity of light in a vacuum, ft/sec
C	Gladstone-Dale constant, $0.117 \text{ ft}^3/\text{slug}$ (eq. (A6))
d	focal-plane aperture diameter, ft (fig. 2)
E	average intensity of light, watts/ft ²
$\frac{E}{E_0}$	intensity or transmittance ratio relative to incident wave
F	focal length of telephotometer, ft (fig. 2)
K	$\left(\frac{4\pi l}{\lambda}\right)^2$ dimensionless grouping indicative of the principal scattering angle for the Markoffian correlation function
K_G	$\left(\frac{2\pi l}{\lambda}\right)^2$ dimensionless grouping indicative of the principal scattering angle for the Gaussian correlation function
l	integral scale of density fluctuations, ft
y	distance through turbulence along path of primary beam, ft
v	velocity of light in air, ft/sec

P	power associated with electromagnetic wave, watts
$R\left(\frac{y}{l}\right)$	correlation function for turbulent fluctuations, dimensionless
α	attenuation coefficient, ft^{-1}
ϵ	absolute dielectric constant, $(\text{emu})^2 \text{lb}^{-1} \text{ft}^{-4} \text{sec}^2$
η	index of refraction $\left(\frac{c}{v}\right)$
θ	scattering angle, radians
θ_M	half-angle subtended by focal-plane aperture, $\tan^{-1} \frac{d}{2F}$, radians
λ	wave length of radiation, ft
μ	absolute permeability, $\text{lb ft}^2 (\text{emu})^{-2}$
ρ	density, slug/ft^3
σ	scattering cross section $(\text{steradian ft})^{-1}$, equation (6)
ω	solid angle corresponding to θ , steradians, $\frac{\omega}{4\pi} = \sin^2 \frac{\theta_M}{2}$
χ	angle between incident electric vector and the scattered ray

Subscripts and Superscripts

Gauss	Gaussian form of correlation function
M	value measured by telephotometer with focal-plane aperture θ_M
s	scattered
T	total
Mark	Markoffian form of correlation function
o	value associated with incident wave
f	value pertaining to measurements in a vacuum
$(\bar{})$	mean value

THEORY

Consideration is directed first toward an application of the pertinent scattering theory to the attenuation of a plane light wave traversing a scattering medium. To this end, consider the telephotometer system shown schematically in figure 1. A uniform, parallel beam of monochromatic light is directed through a turbulent medium, undergoes scattering, and is received by a telephotometer. The scattered field is examined at the focus of the telephotometer by means of calibrated apertures used in conjunction with a photomultiplier tube. The action of the axially symmetric telephotometer of figure 1 is shown schematically in figure 2. The sketch illustrates the propagation path for one of the scattered rays that just impinges on the periphery of the aperture. By the geometry of the system, all rays scattered through angles greater than θ_M are rejected by the instrument. So long as the angle θ_M is sufficiently small, all other rays making this angle with the optical axis will also impinge on the periphery of the aperture. Thus, it is clear that the angle θ in this case is just equal to the angular aperture of the telephotometer, θ_M . The instrument with an aperture θ_M , then, records only those rays scattered through angles that do not exceed θ_M . As the aperture size increases, the telephotometer receives a greater percentage of the scattered power, and the transmittance ratio derived from observations with the telephotometer increases.

For a given focal-plane aperture, the attenuation that occurs as the light beam traverses the turbulent medium is governed by the considerations which follow. It is known that aggregations of particles such as air, smoke in air, and aerosols attenuate a beam of parallel monochromatic radiation in close accord with Lambert's exponential law for homogeneous media (refs. 5, 6, and 7), which states that the percentage of intensity loss is proportional to the distance traversed:

$$\frac{dE}{E} = -\alpha dy \quad (1)$$

The attenuation per foot of penetration, α , is dependent on the wavelength and state of polarization of the incident radiation, the structure of the scattering particles, and the intensity of fluctuations in particle number density. Ordinarily, α is considered as consisting of two distinct parts, an absorption coefficient and a scattering coefficient, to distinguish between the intensity decrease due to disappearance of visible light into radiation at other frequencies (such as heat) and that due to deflection of visible light from the direction of primary propagation. Because visible light is absorbed only very slightly by air, the attenuation per foot in equation (1) can be taken as that due to scattering of the radiation.

An important consequence of the exponential form of Lambert's law is that measurements made on two or more identical scattering layers can be reduced to a unit penetration. If the radiation passes through

n identical layers (or is reflected back along the same path n times), the logarithmic solution of equation (1) for a constant mean scattering coefficient relates the energy loss for a unit penetration to the total loss for n units of penetration by the expression

$$\left(\frac{E}{E_0}\right)_n = \left(\frac{E}{E_0}\right)_1^n \quad (2)$$

where E/E_0 is the transmittance ratio and the subscripts refer to the ratio of the path lengths. For example, if the radiation passes through two identical layers and is measured, the equivalent loss for one layer is obtained by taking the square root of the measured quantity.

Consideration is directed, secondly, to the effects of the turbulent density fluctuations on the attenuation of a light beam that obeys Lambert's law. The structure of the turbulence affects the transmittance ratio by entering into the specification of the attenuation or scattering coefficient, α . Light, considered as an electromagnetic wave, consists of an electric field and a magnetic field which are mutually perpendicular to each other and to the direction of propagation. As the light wave traverses a medium in which the particle number density fluctuates, the interaction of the electrons associated with the particles and the electric field of the wave causes a reradiation of energy to occur. The reradiation is a manifestation of the accelerations of the light wave. The magnitude of the intensity of the scattered rays is expressed as a fractional part of the primary beam intensity

$$P_s = \sigma E_0 \quad (3)$$

The ratio of the scattered power to the incident intensity is denoted by σ , termed the classical scattering coefficient. By definition of σ , the incident radiation falling on a unit area of a surface perpendicular to the primary beam loses a fraction of its energy equal to σ . Thus, in unit time the medium scatters as much radiation as falls on an area equal to σ ; and, for this reason, σ is commonly referred to as the classical cross section for scattering.

In passing through a scattering volume, an electromagnetic wave undergoes scattering in all directions. This physical result is embodied in the concept of the scattering cross section described previously. Specifically, the scattering cross section is defined as the fraction of the incident radiant flux scattered per unit solid angle per unit volume. When a volume containing a turbulent medium is viewed over the solid angle ω_1 to ω_2 , the average attenuation coefficient is found by integrating the scattering cross section over the solid angle range:

$$\alpha = \int_{\omega_1}^{\omega_2} \sigma(\omega) d\omega \quad (4)$$

where $\sigma(\omega)$ represents the scattering cross section in terms of the solid angle ω . For line-of-sight propagation and axially symmetric receiving systems, the solid angle ω is related to the scattering angle θ , measured from the direction of primary propagation, by the identity

$$\frac{\omega}{4\pi} \equiv \sin^2 \frac{\theta}{2} \quad (5)$$

The scattering cross section can be found by solving Maxwell's wave equations in a region characterized by a fluctuating dielectric constant $\epsilon = \bar{\epsilon} + \Delta\epsilon$ with the $\Delta\epsilon$'s correlated in space by a correlation function $R(y/l)$. Booker and Gordon (ref. 1) obtained a specification of the scattering cross section for a turbulence model with isotropic fluctuations and a space correlation that obeys an exponential (Markoffian) decay law. Their result is:

$$\sigma(\theta) = \frac{\overline{\left(\frac{\Delta\epsilon}{\epsilon}\right)^2} \left(\frac{2\pi l}{\lambda}\right)^3 \sin^2 \chi}{\lambda \left[1 + \left(\frac{4\pi l}{\lambda}\right)^2 \sin^2 \frac{\theta}{2}\right]^2} \quad (6)$$

For forward scattering¹ - which occurs at nearly right angles to the incident electric field - the term $\sin^2 \chi$ can be taken as unity. This simplifies the scattering cross section so that

$$\sigma(\theta) = \frac{\frac{1}{8\lambda} \overline{\left(\frac{\Delta\epsilon}{\epsilon}\right)^2} \left(\frac{4\pi l}{\lambda}\right)^3}{\left[1 + \left(\frac{4\pi l}{\lambda}\right)^2 \sin^2 \frac{\theta}{2}\right]^2} \quad (7)$$

From equation (7), it can be seen that the power distribution in the scattered field is symmetrical about the propagation axis and is proportional to the mean square of the dielectric fluctuations, $\overline{(\Delta\epsilon/\epsilon)^2}$, and is also directly proportional to the cube of the integral scale, l . Equation (7) indicates that the transmittance ratio will be considerably affected by relatively small changes in the turbulence structure. The

¹Rigorously, the forward scattering assumption need not be introduced here. Equation (6) is in a form applicable to polarized radiation; but it can be modified to be applicable to unpolarized radiation by replacing the term $\sin^2 \chi$ by $1 + \cos^2 \theta/2$ (ref. 8). For the small angles associated with forward scattering, the latter term is also essentially unity. In the subsequent development, the results are unaltered by the simplification employed here.

turbulent density fluctuations are simply related to fluctuations in the dielectric constant as shown in Appendix A.

The simplification of equation (6) to equation (7) by restricting the discussion to forward scattering can be justified by considering the effects on the cross section of the ratio $4\pi l/\lambda$. By integration of equation (7) over the limits zero to an arbitrary scattering angle θ , the scattered power found within the cone generated by the corresponding solid angle can be obtained. The result of such an integration is:

$$\frac{P}{P_{s,T}} = 1 - \frac{1 - \sin^2 \frac{\theta}{2}}{1 + \left(\frac{4\pi l}{\lambda}\right)^2 \sin^2 \frac{\theta}{2}}$$

The half-power cone, then, occurs for the angle that has

$$\sin \frac{\theta}{2} = \frac{1}{\sqrt{\left(\frac{4\pi l}{\lambda}\right)^2 + 2}}$$

In anticipation of the result, the sine can be replaced by the angle itself and the factor 2 neglected to define the principal scattering cone as one with a half-angle given by:

$$\theta = \frac{\lambda}{2\pi l}$$

Upon taking 1/10 inch as an order of magnitude of the integral scale in a turbulent boundary layer and using 2×10^{-5} inch (5200 Å) for the wave length of light, one finds that the half-power cone has a half-angle of about 2×10^{-5} radians (about 4 seconds of arc). This justifies the assumption that the scattered wave is beamed mainly in the forward direction. The power polar diagram of the scattering cross-section function shown in figure 3 represents the scattering from a unit volume of turbulence and illustrates the corresponding half-power cone described previously.

A general expression can now be derived relating the ratio of the measured intensities that result when two arbitrarily sized apertures that subtend solid angles ω_M and ω_2 ($\omega_2 > \omega_M$) are placed in the focal plane of the telephotometer of figure 1. Equation (1) can be integrated in closed form by utilizing the scattering cross-section expression given by equation (4). Integrating (1) between the limits ω_M to ω_2 and then 0 to y gives

$$\ln \frac{E_2}{E_M} = \int_0^y \int_{\omega_M}^{\omega_2} \sigma \, d\omega \, dy \quad (8)$$

When the second aperture, ω_2 , is selected to be much larger than the aperture that receives the half-power cone, virtually the entire scattered field can be received. For this condition, the upper limit ω_2 (in eq. (8)) can be relaxed to 4π ; and E_2 can be replaced by E_0 , the intensity associated with the incident wave. In practice under conditions of forward scattering, this reference aperture is actually small because the half-power cone itself is small. The integration can be readily accomplished with the aid of the identity relating θ_M and ω_M (eq. (5)) and the scattering cross section of equation (7). After integrating and using the assumption that the integral scale of the fluctuations is independent of depth of penetration, one obtains the equation for the telephotometer:

$$\ln \frac{E_M}{E_0} = -\frac{\pi}{2\lambda} \sqrt{K} \left(\frac{K}{1+K} \right) \frac{\left(1 - \sin^2 \frac{\theta_M}{2} \right)}{\left(1 + K \sin^2 \frac{\theta_M}{2} \right)} \int_0^y \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 dy \quad (9)$$

where

$$K = \left(\frac{4\pi l}{\lambda} \right)^2$$

With some rearrangement, this becomes $\left(\frac{K}{1+K} \pm 1 \right)$:

$$\frac{-1}{\ln \frac{E_M}{E_0}} = \frac{1}{\frac{2\pi^2 l}{\lambda^2} \int_0^y \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 dy} + \frac{1+K}{\frac{2\pi^2 l}{\lambda^2} \int_0^y \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 dy} \tan^2 \frac{\theta_M}{2} \quad (10)$$

This form of the telephotometer equation relates the transmittance ratio E_M/E_0 measured by the telephotometer with a focal-plane aperture θ_M to the turbulence structure characterized by the integral scale and average intensity of turbulent fluctuations. Substantially the same result occurs for the assumption of a Gaussian form for the correlation function (Appendix B).

The form of the telephotometer expression (eq. (10)) is significant. As shown in figure 4, a plot of the transmittance ratio function $-1/\ln(E_M/E_0)$ is linear with the aperture function $\tan^2(\theta_M/2)$. On the linear plot, the mean structure of the turbulence is contained in the slope and intercept - or graphically in the x and y axis intercepts. The x and y axis intercepts define the integral scale and average intensity of turbulent fluctuations, respectively. Thus, a series of

attenuation measurements obtained from photometric surveys of the light field of a given wave length through apertures of varying diameter can yield values of the unknown turbulent parameters.

EXPERIMENTAL RESULTS

Figure 5 shows the extent to which the foregoing analysis is borne out by experiment. The telephotometer arrangement was similar to that shown schematically in figure 1; and the density fluctuations occurred in the two turbulent boundary layers on the side walls of the 1- by 3-foot supersonic wind tunnel no. 1 at the Ames Aeronautical Laboratory. Free-stream conditions were maintained constant and the boundary layers were altered from the normal smooth-wall case both by means of single roughness elements located upstream of the throat and bands of distributed roughness of different extent along the tunnel side walls. Transmittance ratios measured with a set of focal-plane apertures are plotted in the functional form suggested by equation (10) for each boundary-layer configuration. The linear result predicted by the analysis was obtained. For the conditions shown, the integral scales range from 1/10 to 1/25 of the boundary-layer thickness, and the average fluctuation intensities range up to about 5 percent of the mean boundary-layer density. Analogous values of 2/5 and 5 percent for velocity turbulence have been measured in low-speed flows by Klebanoff and Diehl (ref. 9).

DISCUSSION

Over the range of flow conditions tested, the telephotometer data exhibit the functional dependence between transmittance ratio and aperture function predicted by the analysis. This experimental agreement lends weight to the assumptions made in the analysis and allows interpretation of the x and y axis intercepts of equation (10) (fig. 4) in terms of the turbulence parameters. However, the interpretation placed upon the intercepts is influenced by the turbulence model used in the analysis. In particular, the interpretation is affected by the mathematical form of the correlation function.

The specification of the scattering cross section (eq. (6)) is directly influenced by the form of the correlation function. The scattering cross section used in this paper is based upon the Markoffian

correlation function $\left(R(y/l) = e^{-(y/l)}\right)$. This form was selected because

it has been found applicable to forward scattering phenomena in the radio-frequency portion of the spectrum. It should be noted that this form is not necessarily the best for application to light scattering by turbulent boundary layers.

Physically, the correlation function obtained by measurements is characterized by zero slope at the origin which can be ascribed to viscous damping over a region characterized by the microscale of turbulence (ref. 10). The Markoffian function does not possess this property. A

Cauchy form $\left(R(y/l) = 1/[1 + (y/l)^2]^2\right)$ or a Gaussian form $\left(R(y/l) = e^{-(y/l)^2}\right)$

of correlation function is physically better suited because the slope is zero at the origin for these functions. Using the cross section expressions obtained under these two assumptions (Wheelon and Muchmore, ref. 3, p. 1452), one can obtain the equation for the telephotometer for these two forms. For the Cauchy form, the result leads to a functional dependence that is not verified by the optical data. For the Gaussian form, however, the telephotometer expression is essentially identical to equation (10). The differences between the two forms (eqs. (10) and (B5)) are confined to numerical constants affecting the magnitudes of the integral scale and intensity of fluctuations deduced from the data. The interpretations assigned to the x and y axis intercepts are the same for both cases (Markoffian and Gaussian). For a given set of data, such as any one of the configurations shown in figure 5, the Gaussian interpretation of the intercepts leads to values that differ by factors of 2 (approximately) from the corresponding values from the Markoffian assumption:

$$l_{\text{Gauss}} = 2l_{\text{Mark}}$$

$$\left[\int_0^y \overline{\left(\frac{\Delta \epsilon}{\epsilon}\right)^2} dy \right]_{\text{Gauss}} = \frac{1}{\sqrt{\pi}} \left[\int_0^y \overline{\left(\frac{\Delta \epsilon}{\epsilon}\right)^2} dy \right]_{\text{Mark}}$$

An experiment designed to measure the integral scale and intensity simultaneously by an independent method is required to show which assumption is the more applicable to the turbulent boundary layer.

CONCLUDING REMARKS

To summarize, it has been shown analytically that plane light waves traversing a turbulent boundary layer undergo forward scattering which introduces a reduction in the transmittance ratio. Further, it has been demonstrated that photometric data obtained by measurements taken in the scattered field are in functional agreement with the predictions of the analysis. The interpretation of experimental data in terms of the turbulence parameters, however, depends upon the turbulence model employed and is in doubt by at least a factor of 2 for the models studied.

The principal advantage of the optical method described here is that the phenomenon of scattering is dependent only on the density fluctuations.

The measurements, therefore, are not hampered by a mixed response to several fluctuating variables as is the hot-wire anemometer. Other advantageous features are that no probe and a minimum of electronic circuitry is required.

The greatest limitation of the method is that, inherently, the results obtained are integrated averages over the volume of turbulence irradiated by the incident beam. Also, the possibility exists that for conditions of high losses, the results can become severely contaminated by the effects of secondary scattering, which were ignored throughout the analysis.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

RELATIONSHIP BETWEEN DIELECTRIC FLUCTUATIONS

AND DENSITY FLUCTUATIONS FOR AIR

For the system of units force, length, time, and electromagnetic unit of charge (emu), the wave velocity in any medium is given by (ref. 8)

$$\frac{1}{\sqrt{\mu\epsilon}} = v \quad (A1)$$

The velocity of wave propagation in a vacuum is the speed of light

$$\frac{1}{\sqrt{\mu_P\epsilon_P}} = c \quad (A2)$$

The refractive index of a medium is defined by the ratio of the speed of light in a vacuum to the speed of light in the medium:

$$\eta = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_P\epsilon_P}} \quad (A3)$$

For gases, the permeability to electromagnetic radiation is essentially that for a vacuum; the dielectric constant, however, differs by small, measurable amounts. For gases, then, the refractive index is given to good approximation by:

$$\eta = \sqrt{\frac{\epsilon}{\epsilon_P}} \quad (A4)$$

Squaring the logarithmic differential of equation (A4) yields the relation between fluctuations in the refractive index and the dielectric constant of the medium:

$$4\left(\frac{d\eta}{\eta}\right)^2 = \left(\frac{d\epsilon}{\epsilon}\right)^2 \quad (A5)$$

For air, the refractive index is given by

$$\eta = 1 + C\rho \quad (A6)$$

where C is the Gladstone-Dale constant, $0.117 \text{ ft}^3/\text{slug}$ at $5200 \text{ \AA}$. Combining equations (A5) and (A6) gives the relation between dielectric fluctuations and density fluctuations for air:

$$\left(\frac{d\epsilon}{\epsilon}\right)^2 \doteq 4C^2(d\rho)^2 \quad (\text{A7})$$

APPENDIX B

TELEPHOTOMETER EQUATION FOR THE CASE OF A

GAUSSIAN CORRELATION COEFFICIENT

The scattering cross section derived under the assumption of a Gaussian form of the correlation coefficient is (Wheelon and Muchmore, p. 1452, ref. 3)

$$\sigma = \frac{\sqrt{\pi}}{8\lambda} K_G^{3/2} \left(\frac{\Delta\epsilon}{\epsilon} \right)^2 e^{-K_G \frac{\omega}{4\pi}} \quad (B1)$$

where

$$K_G = \left(\frac{2\pi l}{\lambda} \right)^2 \quad (B2)$$

(The relation between the solid angle and the scattering angle is the identity given previously (eq. (5)) $\omega/4\pi \equiv \sin^2(\theta/2)$). For cases of forward scattering wherein essentially all of the scattered energy can be found within very small solid angles, the upper limit in equation (8) can be taken as 4π as before. Carrying out the integration of equation (8) using equations (B1) and (B2) and noting that $e^{-K_G} \ll e^{-K_G(\omega_M/4\pi)}$, one obtains

$$\frac{-1}{\ln \frac{E_M}{E_O}} \doteq \frac{1}{\frac{\sqrt{\pi^3} \sqrt{K_G}}{2\lambda} \int_0^y \left(\frac{\Delta\epsilon}{\epsilon} \right)^2 dy} e^{K_G \frac{\omega_M}{4\pi}} \quad (B3)$$

For the small angles encountered in forward scattering measurements, the first two terms of the exponential expansion can be used to obtain:

$$\frac{-1}{\ln \frac{E_M}{E_O}} \doteq \frac{1}{\sqrt{\pi} \frac{\pi^2 l}{\lambda^2} \int_0^y \left(\frac{\Delta\epsilon}{\epsilon} \right)^2 dy} \left(1 + K_G \frac{\omega_M}{4\pi} \right) \quad (B4)$$

or

$$\ln \frac{E_M}{E_0} = \frac{1}{2} \frac{\sqrt{\pi}}{\lambda^2} \frac{1}{\int_0^y \left(\frac{\Delta \epsilon}{\epsilon}\right)^2 dy} + \frac{1}{2\sqrt{\pi}} \frac{K}{\lambda^2} \frac{1}{\int_0^y \left(\frac{\Delta \epsilon}{\epsilon}\right)^2 dy} \tan^2 \frac{\theta_M}{2} \quad (B5)$$

where the sine has been replaced by the tangent to correspond to equation (10). Comparison of equations (B5) and (10) shows that the forms are the same; the difference between the two results lies in the constant coefficients $\sqrt{\pi}/2$ and $2\sqrt{\pi}$ that are found in equation (B5). The term K is essentially equivalent to $1 + K$ because $K \gg 1$.

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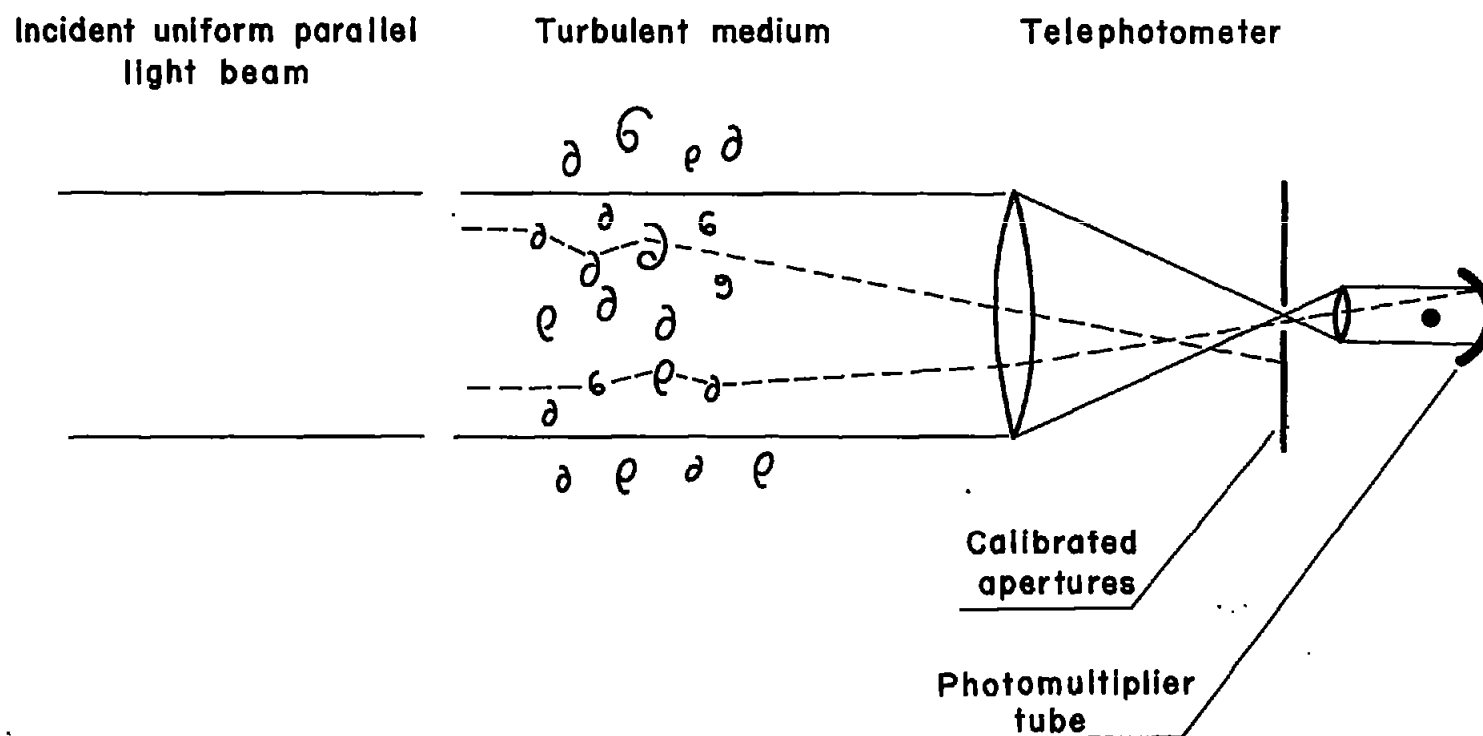


Figure 1.- Experimental arrangement for viewing a light beam scattered by turbulent density fluctuations.

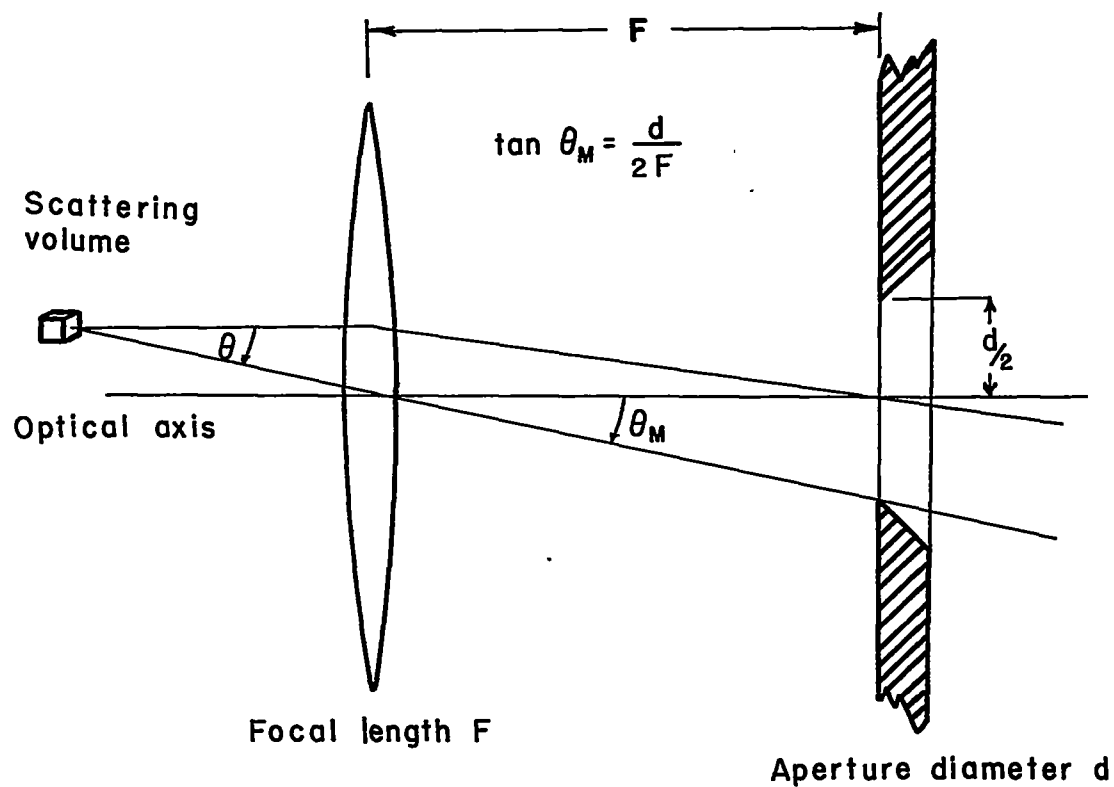


Figure 2.- Schematic view illustrating that only those rays that are scattered through angles that are less than the half-angle of the focal-plane aperture are received by the telephotometer.

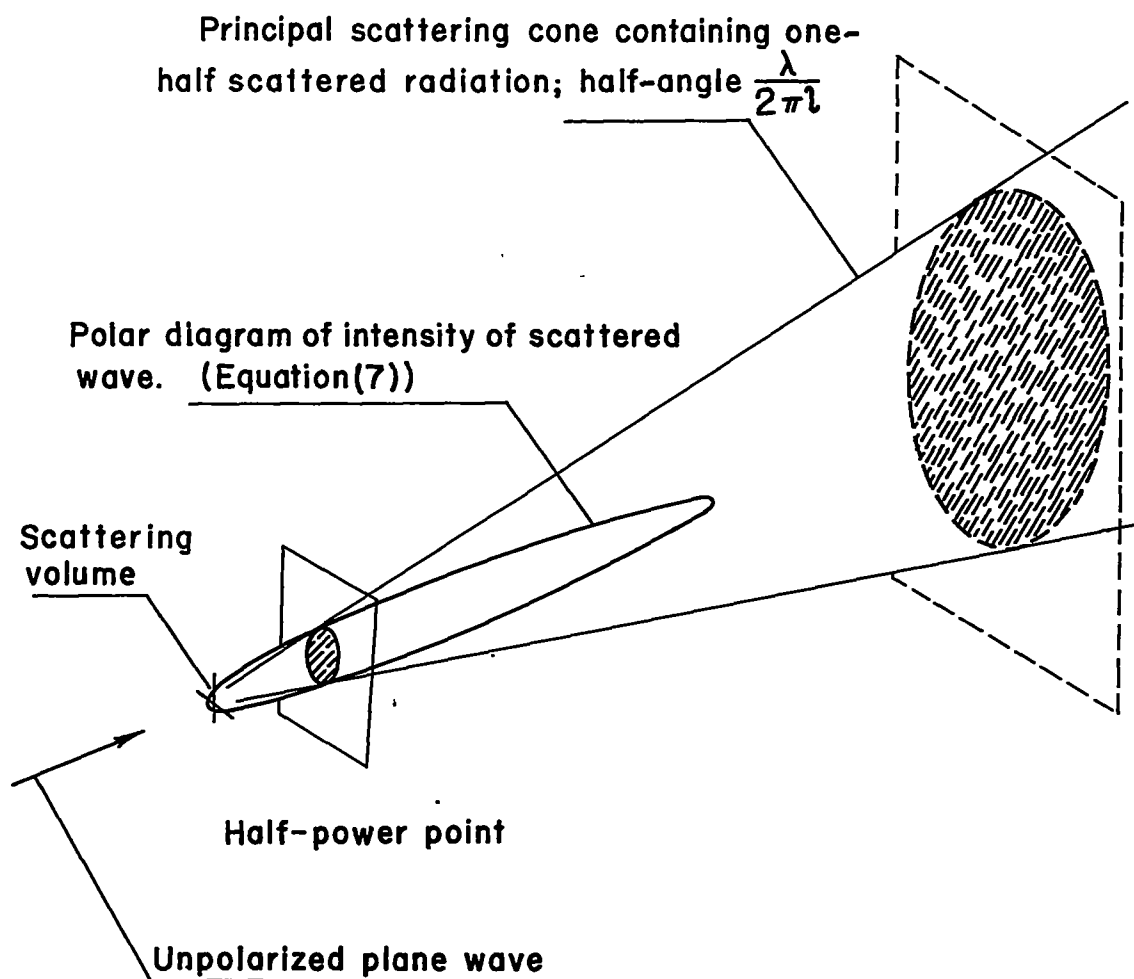


Figure 3.- Principal scattering cone for an incident plane wave for the scattering cross section given by Booker and Gordon (angles greatly enlarged).

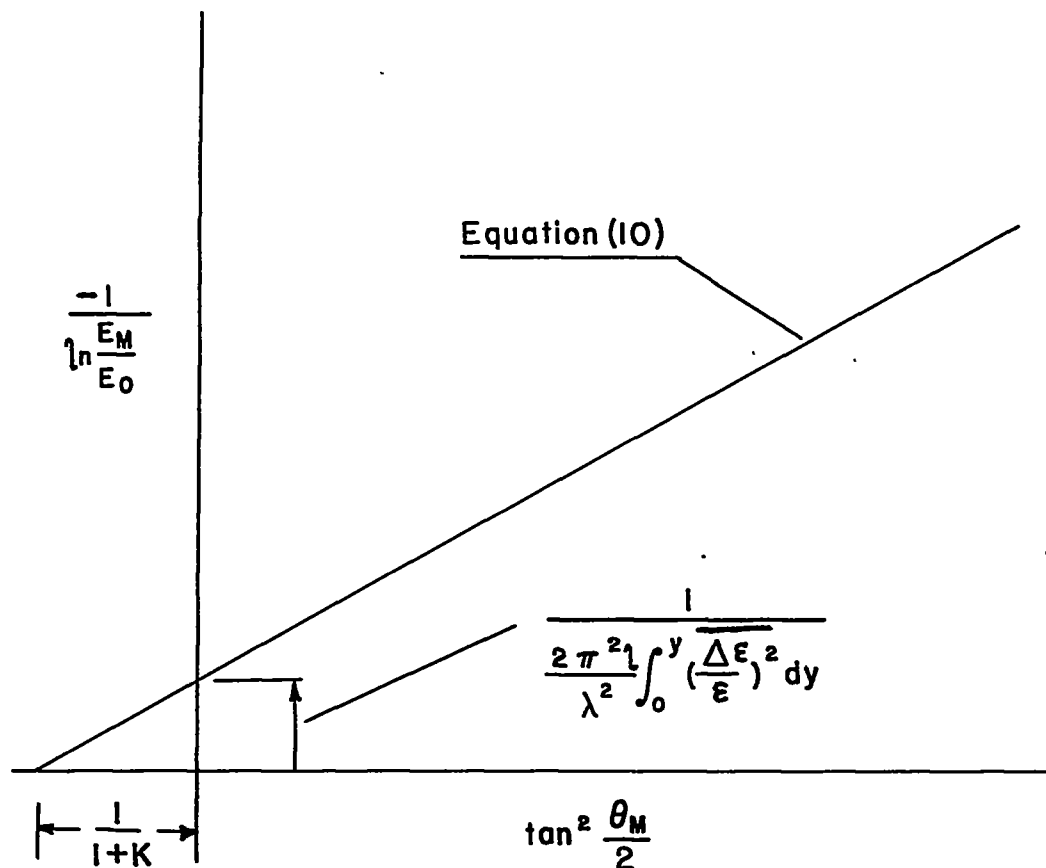


Figure 4.- Predicted linear form of the telephotometer equation showing the average integral scale and intensity of turbulent fluctuations in terms of the x and y axis intercepts.

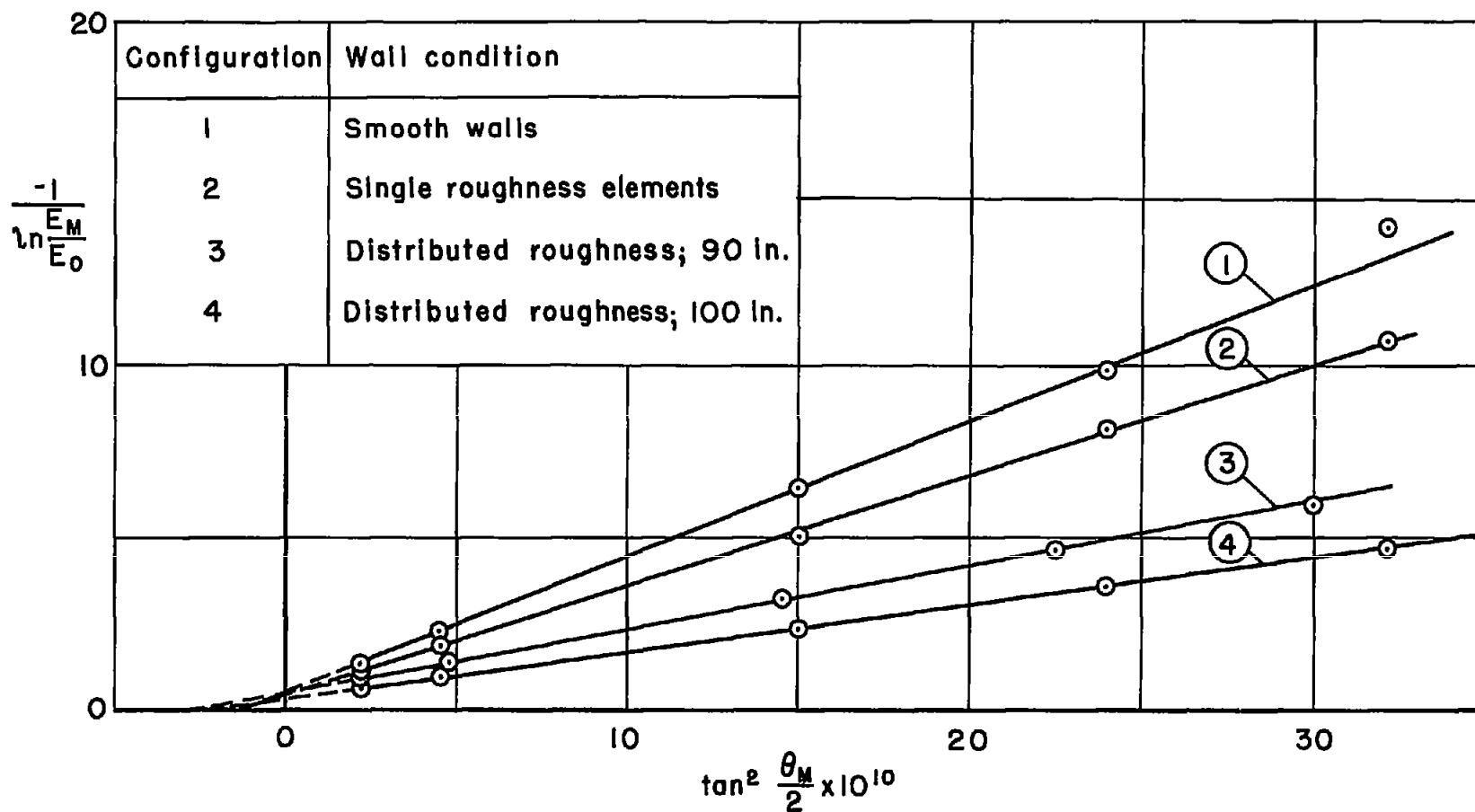


Figure 5.- Demonstration of the linear dependence obtained for telephotometer measurements in the scattered light field for fixed free-stream conditions and variable boundary-layer thickness.